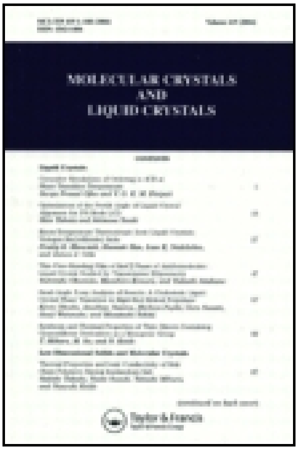

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ANALYTICAL SOLUTION FOR THERMAL CONVECTION IN A LAYER OF LIQUID CRYSTAL WITH FREE SURFACE

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An Oberbeck-Bussinesq type model for convection in isothermically incompressible liquid crystals (nematics and cholesterics) is proposed. For its derivation, the nonequilibrium thermodynamics and asymptotic methods are employed. The model is based on the Leslie-Ericksen-Parodi (LEP) continuum theory [1–4] taken with some minor modifications. The problem of the thermal gravity convection in a plane horizontal layer with 'free undeformable' boundary and under the action of external magnetic or electric field is considered. Solution of this problem is obtained and presented in an explicit analytical form. Existence of the explicit analytical solution is very useful, since it allows us to investigate the influence of various parameters (Leslie coefficients, Frank moduli, anisotropic thermo-diffusivity, etc.) at the onset of convection.

Keywords: analytical solution; cholesterics; nematics; thermal convection

Similar problems have been treated numerically (see e.g. [5–8]) for problems with non-slip boundary conditions. The results have been compared with experiments (see e.g. [9,10]). However it is very difficult to analyze the dependence of the numerical solution and critical values of parameters when more than ten physical parameters (coefficients) are involved. The 'undeformable free' boundary condition considered in this paper may look rather artificial, however in hydrodynamic convection theory they play important role as they provide the 'test' solutions (see e.g. [12] for the Rayleigh-Benard problem). It is known, that in the Rayleigh-Benard

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problem the qualitative dependence of neutral curves and critical values on the physical parameters for solid and 'free undeformable' boundaries are similar while the critical Rayleigh number for solid boundary is approximately three times higher.

1. BASIC EQUATIONS

The Oberbeck-Boussinesq type model of the thermogravitational convection for incompressible liquid crystal (LC) in dimensionless variables has the form [11] (the term 'incompressible' means that the density of the medium depends only on the temperature):

$$\operatorname{div}\mathbf{v}=0,\ \frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla,\ \mathbf{n}\cdot\mathbf{n}=1 \tag{1}$$

$$\frac{d\mathbf{v}}{dt}=-\nabla p+\nabla\cdot\Pi+\frac{1}{2}\operatorname{curl}(\mathbf{n}\times(\mathbf{h}+\mathbf{G}))-\mathbf{h}\times\operatorname{curl}\mathbf{n}-(\mathbf{h}\cdot\nabla)\mathbf{n}+T\mathbf{k},$$

$$\gamma_1\left(\frac{d\mathbf{n}}{dt}-\boldsymbol{\Omega}\times\mathbf{n}\right)-\gamma_2\mathbf{n}\times(\mathbf{n}\times(\mathbf{D}\cdot\mathbf{n}))+\lambda_1\nabla T\times\mathbf{n}=\mathbf{h}+\mathbf{G},$$

$$\frac{dT}{dt}+\operatorname{div}\{-\delta_1\nabla T-\delta_2(\mathbf{n}(\mathbf{n}\cdot\nabla T)-\nabla T)+\zeta_1\mathbf{N}\times\mathbf{n}+2\zeta_2\mathbf{n}\times(\mathbf{D}\cdot\mathbf{n})\}=0,$$

$$\begin{aligned}\Pi=&\alpha_4\mathbf{D}+\alpha_1(\mathbf{n}\cdot\mathbf{D}\cdot\mathbf{n})\mathbf{n}\otimes\mathbf{n}+\frac{1}{2}(\alpha_2+\alpha_3)(\mathbf{N}\otimes\mathbf{n}+\mathbf{n}\otimes\mathbf{N})\\&+\frac{1}{2}(\alpha_5+\alpha_6)(\mathbf{n}\otimes(\mathbf{D}\cdot\mathbf{n})+(\mathbf{D}\cdot\mathbf{n})\otimes\mathbf{n})\\&+\lambda_2(\mathbf{n}\otimes(\mathbf{n}\times\nabla T)+(\mathbf{n}\times\nabla T)\otimes\mathbf{n}),\end{aligned}$$

$$\mathbf{h}=(\mathbf{n}\times(K_1\mathbf{h}^s+K_2\mathbf{h}^t+K_3\mathbf{h}^b+K_0\mathbf{h}^\beta))\times\mathbf{n},$$

$$\mathbf{h}^b=\operatorname{curl}(\mathbf{n}\times(\mathbf{n}\times\operatorname{curl}\mathbf{n}))+(\mathbf{n}\times\operatorname{curl}\mathbf{n})\times\operatorname{curl}\mathbf{n},\ \mathbf{h}^\beta=-2\operatorname{curl}\mathbf{n},$$

$$\mathbf{h}^t=-\operatorname{curl}(\mathbf{n}(\mathbf{n}\cdot\operatorname{curl}\mathbf{n}))-(\mathbf{n}\cdot\operatorname{curl}\mathbf{n})\operatorname{curl}\mathbf{n},\ \mathbf{h}^s=\nabla\operatorname{div}\mathbf{n},$$

$$\mathbf{N}=\frac{d\mathbf{n}}{dt}-\boldsymbol{\Omega}\times\mathbf{n},\ \boldsymbol{\Omega}=\frac{1}{2}\operatorname{curl}\mathbf{v},\ \mathbf{D}=\frac{1}{2}\{(\nabla\mathbf{v})+(\nabla\mathbf{v})^*\}.$$

Here, $\mathbf{v}$ is the velocity, p is the pressure, T is the temperature difference, $\mathbf{n}$ is the director, Π is the symmetric part of stress tensor, related to the motion of the medium, $\mathbf{k}$ is the unit vector along the vertical x_3 axis in the direction opposite to the gravity force, $\mathbf{h}$ is the molecular field, $\mathbf{D}$ is the rate of strain tensor, $\alpha_1,\ldots,\alpha_6$ are scalar Leslie coefficients, γ_1, γ_2 are scalar rotational viscosity coefficients, δ_1,δ_2 are scalar thermodiffusivity coefficients, K_1,K_2,K_3, are scalar Frank moduli, λ_1,λ_2 are *pseudoscalar* coefficients, which characterize the influence of temperature gradients on the director field and stress tensor, ζ_1, ζ_2 are *pseudoscalar* coefficients, which characterize the influence of fluid and director motion on the heat flux, K_0 is the *pseudoscalar* coefficient, which defines the pitch of spiral structures in cholesterics ($K_0 = 2\pi K_2/p_0$, where p_0 is the pitch), **G** is the external torque. The torque **G** for the magnetic and electric fields is given as

$$\mathbf{G} = \chi_\alpha(\mathbf{n}\cdot\mathbf{H})(\mathbf{H} - \mathbf{n}(\mathbf{n}\cdot\mathbf{H})) + \varepsilon_\alpha(\mathbf{n}\cdot\mathbf{E})(\mathbf{E} - \mathbf{n}(\mathbf{n}\cdot\mathbf{E})), \tag{2}$$

where χ_a, ε_a are the magnetic anisotropy and the dielectric anisotropy (other terms in use are the magnetic susceptibility and the electric susceptibility). We have assumed that there are no electrical charges in the medium. We have also assumed that the specific volume of the medium is represented by a linear function of temperature: $V(T) = V_0(1 + \beta T)$, where $V = 1/\rho$ is the specific volume, ρ is the density, β is the heat expansion coefficient. Equation (1) have been obtained via a rigorous limit procedure when $\beta \to 0$, under the usual assumption that the gravity force has the order of magnitude of $O(1/\beta)$. Those equations will be applicable at least far from the temperature phase transition points.

Let us notice some features of the system (1), (2). We introduced the 'normalized' molecular field **h**, and torque **G** which satisfy the conditions $\mathbf{h}\cdot\mathbf{n} = \mathbf{0}, \mathbf{G}\cdot\mathbf{n} = \mathbf{0}$. They reflect the fact that the molecular field and the external torque can only rotate the director without changing its length. The director equation is transformed with the use of conditions $\mathbf{h}\cdot\mathbf{n} = \mathbf{0}$, $\mathbf{G}\cdot\mathbf{n} = \mathbf{0}$ and is automatically in agreement with the equality $\mathbf{n}\cdot\mathbf{n} = 1$. The 'normalized' form also makes easier general mathematical investigations. We have regrouped terms in the stress tensor: the terms related to the nondissipative effects (corresponding to the elastic deformation of the continuum) are presented explicitly. Notice, that the presence of *pseudoscalar* coefficients is typical for cholesterics and allows one to explain the Lehmann effect. In the case of nematics pseudoscalar coefficients should be omitted:

$$K_0 = 0,\ \lambda_1 = 0,\ \zeta_1 = 0,\ \lambda_2 = 0,\ \zeta_2 = 0. \tag{3}$$

2. FORMULATION OF PROBLEM

Consider the problem of thermal convection for nematics LC in a plane horizontal layer $(-\infty < x_1, x_2 < +\infty, 0 \leq x_3 \leq 1)$. The boundary conditions take the form:

$$\mathbf{n}|_{x_3=0,1} = \mathbf{k},\ T|_{x_3=0}T^0,\ T|_{x_3=1} = T^1,\ \mathrm{v}_3|_{x_3=0,1} = 0,\ \sigma_{i3}|_{x_3=0,1} = 0, \tag{4}$$

where T^0, T^1 are given constants, $\sigma_{i3}, (i = 1, 2)$ are tangent components of the stress tensor at the boundaries. The expression for the stress tensor is:

$$\begin{aligned}\sigma_{ik} = &- p\delta_{ik} + \Pi_{ik} + \frac{1}{2}\{n_i(h_k + G_k) - (h_i + G_i)n_k\} \\ &- \{K_0 n_s \varepsilon_{skm}\partial_i n_m + K_1 \partial_i n_k \operatorname{div}\mathbf{n} + K_2(\partial_k n_m - \partial_m n_k)\partial_i n_m \\ &+ (K_3 - K_2) n_s n_k \partial_s n_m \partial_i n_m\}\end{aligned}$$

Accepted conditions mean that the temperature on each boundary is maintained constant; the director field is normal to the layer surfaces (homeotropic orientation) and cannot be changed. The last two conditions (4) correspond to the classical 'undeformed free' boundary. We also suppose that external electric field is perpendicular to the layer ($\mathbf{E} = E_0\mathbf{k}$). The magnetic field is omitted for the sake of simplicity and can be recovered easily. There is an equilibrium solution of the problem (1)–(4):

$$\mathbf{v}^0 = 0,\ \mathbf{n}^0 = \ \mathbf{k},\ T_0 = ax_3 + b,\ p_0 = p_0(x_3),\ T_0' = a = T^1 - T^0,\ b = T^0. \tag{5}$$

3. LINEARIZED STABILITY PROBLEM

We look for a solution of the problem (1)–(4) in the form

$$\mathbf{v} = \mathbf{v}^0 + \delta\mathbf{v} = \delta\mathbf{v},\ \mathbf{n} = \mathbf{n}^0 + \delta\mathbf{n},\ T = T_0 + \delta T,\ p = p_0 + \delta p.$$

where $\delta\mathbf{v},\ \delta\mathbf{n},\ \delta T,\ \delta p$ are linear perturbations of the equilibrium.

The basic equations, linearized at the solution (5) after taking into account the relation $\mathbf{k}\cdot\delta\mathbf{n} = 0$ can be written as:

$$\begin{aligned}-\partial_t\Delta\delta \mathrm{v}_3 = &-\frac{1}{2}\alpha_4\Delta^2\delta\mathrm{v}_3 - \alpha_1\Delta_0{\partial_3}^2\delta\mathrm{v}_3 - \frac{1}{4}(\alpha_5+\alpha_6)\Delta^2\delta\mathrm{v}_3 \\ &+ \frac{1}{2}(\alpha_2+\alpha_3)({\partial_3}^2 - \Delta_0)\left(\partial_t \operatorname{div}\delta\mathbf{n} + \frac{1}{2}\Delta\delta\mathrm{v}_3\right) \\ &- \frac{1}{2}\Delta\big((K_3{\partial_3}^2 + K_1\Delta_0 - \varepsilon_\alpha {E_0}^2)\operatorname{div}\delta\mathbf{n}\big) - \Delta_0\delta T.\end{aligned} \tag{6}$$

$$\begin{aligned}\gamma_1\left(\partial_t\operatorname{div}\delta\mathbf{n} + \frac{1}{2}\Delta\delta\mathrm{v}_3\right) + \frac{1}{2}\gamma_2(\Delta_0 - {\partial_3}^2)\delta\mathrm{v}_3 \\ = (K_1\Delta_0 + K_3{\partial_3}^2 - \varepsilon_\alpha {E_0}^2)\ \operatorname{div}\ \delta\mathbf{n},\end{aligned} \tag{7}$$

$$\partial_t\delta T + T_0'\delta\mathrm{v}_3 = (\delta_1 - \delta_2)\Delta\delta T + \delta_2(T_0'\operatorname{div}\delta\mathbf{n} + {\partial_3}^2\delta T), \tag{8}$$

$$2\partial_t\delta\mathbf{\Omega}_3 = \alpha_4\Delta\delta\mathbf{\Omega}_3 + \frac{1}{2}(\alpha_2+\alpha_3)\partial_3(\partial_t \text{curl}_3\delta\mathbf{n} - \partial_3\delta\mathbf{\Omega}_3) + \frac{1}{2}(\alpha_5+\alpha_6)\partial_3{}^2\delta\mathbf{\Omega}_3 - \frac{1}{2}\partial_3\{(K_3\partial_3{}^2 + K_2\Delta_0 - \varepsilon_\alpha E_0{}^2)\text{curl}_3\delta\mathbf{n}\} \quad (9)$$

$$\gamma_1\partial_t \text{curl}_3\delta\mathbf{n} - (\gamma_1-\gamma_2)\partial_3\delta\mathbf{\Omega}_3 = (K_3\partial_3{}^2 + K_2\Delta_0 - \varepsilon_\alpha E_0{}^2)\ \text{curl}_3\ \delta\mathbf{n}, \quad (10)$$

where $\Delta_0 = \partial_1{}^2 + \partial_2{}^2$ is the two dimensional Laplace operator, $2\delta\mathbf{\Omega}_3 = \text{curl}_3\ \delta\mathbf{v}$, $\text{curl}_3\ \delta\mathbf{n} = \mathbf{K}\cdot\text{curl}\ \delta\mathbf{n}$, $\partial_i = \partial/\partial x_i$, $\partial_t = \partial/\partial t$ etc. Notice, that this system of equations can be represented as two subsystems: the first one (6)–(8) for unknown functions δv_3, $\text{div}\,\delta\mathbf{n}$, δT, and the second one (9), (10) describing $\delta\mathbf{\Omega}_3$, $\text{curl}_3\delta\mathbf{n}$. Boundary conditions for equations (6)–(8) at $x_3 = 0, x_3 = 1$ are the following:

$$\delta\text{v}_3 = 0,\ \partial_3{}^2\delta\text{v}_3 = 0,\ \text{div}\,\delta\mathbf{n} = 0,\ \delta T = 0. \quad (11)$$

Boundary conditions for (9), (10) at the same boundaries $x_3 = 0, x_3 = 1$ are:

$$\text{curl}_3\delta\mathbf{n} = 0,\ \partial_3\delta\mathbf{\Omega}_3 = 0 \quad (12)$$

4. NORMAL MODES

We look for solutions of (6)–(12) being periodic in horizontal coordinates with values of periods $2\pi/k_1$, $2\pi/k_2$. This kind of representation in hydrodynamic stability is called 'normal modes'

$$\{\delta\mathbf{v},\ \delta\mathbf{n},\ \delta T\} = \{\overline{\delta\mathbf{v}}(x_3),\ \overline{\delta\mathbf{n}}(x_3),\ \overline{\delta T}(x_3)\}\exp[\lambda t + i(k_1x_1 + k_2x_2)] \quad (13)$$

where k_1, k_2 are the components of the wave vector, λ is the time increment, and $\overline{\delta\mathbf{v}}(x_3)$, $\overline{\delta\mathbf{n}}(x_3)$, $\overline{\delta T}(x_3)$ are the amplitudes of perturbations.

After substitution of (13) into (6)–(12), one can derive the boundary value problem for correspondent system of ODEs. A solution of that system automatically satisfying the boundary conditions (11), (12) is

$$\{\delta\text{v}_3,\ \text{div}\,\delta\mathbf{n}, \delta T, \partial_3\mathbf{\Omega}_3, \text{curl}_3\delta\mathbf{n}\} = \{w, \eta, \mathbf{\Theta}, \chi, \xi\}\sin(m\pi x_3) \times \exp[\lambda t + i(k_1x_1 + k_2x_2)]$$

where m is an integer and w, $\mathbf{\Theta}$, η, ξ, χ are some constants. In order to find those constants one can derive a system of algebraic equations, which follows from (6)–(10) after formal substitutions:

$$\partial_3{}^2 \to -m^2\pi^2,\ \partial_t \to \lambda,\ \Delta_0 \to -k^2,\ \Delta \to -(k^2 + m^2\pi^2),\ k^2 = k_1{}^2 + k_2{}^2,$$
$$\delta\text{v}_3 \to w,\ \text{div}\,\delta\mathbf{n} \to \eta,\ \text{curl}_3\delta\mathbf{n} \to \xi,\ \delta\mathbf{\Omega}_3 \to \chi,\ \delta T \to \mathbf{\Theta}.$$

These substitutions give two systems of linear equations:

$$\begin{aligned} M_{11}w + M_{12}\eta + M_{13}\Theta &= 0, \\ M_{21}w + M_{22}\eta + M_{23}\Theta &= 0, \\ M_{31}w + M_{32}\eta + M_{33}\Theta &= 0. \end{aligned} \tag{14}$$

$$A_{11}\chi + A_{12}\xi = 0,\ A_{21}\chi + A_{22}\xi = 0. \tag{15}$$

Matrix M_{ik}, can be written as

$$\begin{aligned} &M_{11} = -\lambda(m^2\pi^2 + k^2) + m_{11},\ M_{12} = \frac{1}{2}(\alpha_2 + \alpha_3)(-m^2\pi^2 + k^2)\lambda + m_{12}, \\ &M_{13} = k^2,\ M_{21} = m_{21},\ M_{22} = \gamma_1\lambda + m_{22},\ M_{23} = 0, \\ &M_{31} = -T_0',\ M_{32} = \delta_2 T_0',\ M_{33} = -\lambda + m_{33}, \end{aligned}$$

where

$$\begin{aligned} m_{11} =& \frac{1}{4}(\alpha_2 + \alpha_3)(m^4\pi^4 - k^4) \\ &- \left(\frac{1}{2}\alpha_4 + \frac{1}{4}(\alpha_5 + \alpha_6)\right)(m^2\pi^2 + k^2)^2 - \alpha_1 k^2 m^2\pi^2, \\ m_{22} =& K_3 m^2\pi^2 + K_1 k^2 + \varepsilon_\alpha {E_0}^2,\ m_{33} = -\delta_1(m^2\pi^2 + k^2) + \delta_2 k^2, \\ m_{21} =& -\frac{1}{2}\gamma_1(m^2\pi^2 + k^2) + \frac{1}{2}(-k^2 + m^2\pi^2), \\ m_{21} =& -\frac{1}{2}(m^2\pi^2 + k^2)(K_3 m^2\pi^2 + K_1 k^2 + \varepsilon_\alpha {E_0}^2). \end{aligned}$$

Matrix A_{ik} is given as:

$$\begin{aligned} A_{11} &= \gamma_1\lambda + (K_3 m^2\pi^2 + K_2 k^2 + \varepsilon_\alpha E_0^2),\ A_{12} = \gamma_2 - \gamma_1, \\ A_{21} &= -\frac{1}{2}(\alpha_2 + \alpha_3)m^2\pi^2\lambda - \frac{1}{2}m^2\pi^2(K_3 m^2\pi^2 + K_2 k^2 + \varepsilon_\alpha E_0^2), \\ A_{22} &= -\alpha_4(m^2\pi^2 + k^2) - 2\lambda - \frac{1}{2}(\alpha_5 + \alpha_6 - \alpha_2 - \alpha_3)m^2\pi^2. \end{aligned}$$

5. SPECTRAL PROBLEM

Dependence of the exponent λ on the parameters can be obtained after evaluation of the determinants for (14), (15). Recall that Re $\lambda < 0$ corresponds to stable solutions, the case Re $\lambda = 0$, Im $\lambda \neq 0$ describes to oscillatory instability, and Re $\lambda > 0$ gives monotonically unstable solutions. In order to investigate the sign of λ, one should use the inequalities

$K_1 \geq 0,\ K_2 \geq 0,\ K_3 \geq 0$ and constraints for kinetic coefficients

$$\gamma_1 \geq 0,\ \alpha_4 \geq 0,\ (2\alpha_1 + 3\alpha_4 + 2\alpha_5 + 2\alpha_6) \geq 0,\ (2\alpha_4 + \alpha_5 + \alpha_6) \geq 0,$$
$$4\gamma_1(2\alpha_4 + \alpha_3 + \alpha_6) \geq (\alpha_2 + \alpha_3 + \gamma_2)^2,\ \delta_1 \geq 0,\ (\delta_1 - \delta_2) \geq 0$$

which follow from the fundamental fact that the entropy source must be non-negative. One can show, that (in the case $\gamma_1 > 0$, $\varepsilon_a \geq 0$), nontrivial solutions of (15) can exist only when Re $\lambda < 0$. It means that nontrivial solutions of the problem (9), (10), (12), are always stable and rapidly decaying. In that case unstable solutions of the full system can exist only in the case of the trivial solution of the problem (9), (10), (12), i.e. when $\mathrm{curl}_3\delta\mathbf{n} = 0$, $\mathrm{curl}_3\delta\mathbf{v} = 0$. In other words, for investigations of instability (at least when $\varepsilon_a \geq 0$) it is sufficient to solve only problem (14).

In the next two sections we present and analyse the neutral curves for monotonic and oscillatory instabilities. Particular values of physical parameters are taken for the MBBA. Unfortunately there are still at least two different sets of parameters for MBBA in literature (see Table I). The first column of the Table I corresponds to the data from [5,13–15] (we denote it as MBBA(1)), the second one presents the data of [16–19] (called MBBA(2)). One can see that although in average those sets are rather close to each other, there are significant differences in values of such parameters as $K_1,\ K_2,\ K_3,\ \alpha_1,\ \alpha_5$. All neutral curves presented below correspond to the special case of zero electric field. It is very surprising that calculations show that critical parameters for the onset of convection are the same, despite of the fact that neutral curves are different.

TABLE I Physical Parameters of MBBA

	MBBA (1)	MBBA (2)
$\rho,\ kg \cdot m^{-3}$	$1.022 \cdot 10^3$	$1.022 \cdot 10^3$
$\beta,\ K^{-1}$	$1.000 \cdot 10^{-3}$	$1.000 \cdot 10^{-3}$
$K_1 \cdot \rho^{-1},\ m^4 \cdot s^{-2}$	$5.30 \cdot 10^{-15}$	$6.66 \cdot 10^{-15}$
$K_2 \cdot \rho^{-1},\ m^4 \cdot s^{-2}$	$2.20 \cdot 10^{-15}$	$4.20 \cdot 10^{-15}$
$K_3 \cdot \rho^{-1},\ m^4 \cdot s^{-2}$	$7.45 \cdot 10^{-15}$	$8.61 \cdot 10^{-15}$
$\alpha_1 \cdot \rho^{-1},\ m^2 \cdot s^{-1}$	$6.5 \cdot 10^{-6}$	$-18.1 \cdot 10^{-6}$
$\alpha_2 \cdot \rho^{-1},\ m^2 \cdot s^{-1}$	$-77.5 \cdot 10^{-6}$	$-110.4 \cdot 10^{-6}$
$\alpha_3 \cdot \rho^{-1},\ m^2 \cdot s^{-1}$	$-1.2 \cdot 10^{-6}$	$-1.1 \cdot 10^{-6}$
$\alpha_4 \cdot \rho^{-1},\ m^2 \cdot s^{-1}$	$83.2 \cdot 10^{-6}$	$82.6 \cdot 10^{-6}$
$\alpha_5 \cdot \rho^{-1},\ m^2 \cdot s^{-1}$	$46.3 \cdot 10^{-6}$	$77.9 \cdot 10^{-6}$
$\alpha_6 \cdot \rho^{-1},\ m^2 \cdot s^{-1}$	$-34.4 \cdot 10^{-6}$	$-33.6 \cdot 10^{-6}$
$\gamma_1,\ m^2 \cdot s^{-1}$	$76.3 \cdot 10^{-7}$	$109.3 \cdot 10^{-7}$
$\gamma_2,\ m^2 \cdot s^{-1}$	$-78.7 \cdot 10^{-7}$	$-111.5 \cdot 10^{-7}$
$\delta_1,\ m^2 \cdot s^{-1}$	$1.54 \cdot 10^{-7}$	$1.54 \cdot 10^{-7}$
$\delta_2,\ m^2 \cdot s^{-1}$	$0.61 \cdot 10^{-7}$	$0.61 \cdot 10^{-7}$

6. MONOTONIC INSTABILITY

Dependence of critical temperature gradient T_0' for monotonic instability on the parameters of the problem can be obtained via evaluation of determinant for (14) at $\lambda = 0$.

Usually, in convection theory, the Rayleigh number (not the temperature gradient) is in use. Let us introduce the following Rayleigh number (see e.g.[10])

$$Ra = \frac{2\beta T_0'}{\alpha_4 \delta_1}, \quad \text{grad} T_{\text{dim}} = \frac{\alpha_4^* \delta_1^*}{2\beta_* g_* L_*^3} Ra, \tag{16}$$

where β, α_4, δ_1, β_*, α_4^*, δ_1^* are the dimensionless and dimensional coefficients of heat expansion, viscosity and thermodiffusivity; g_*, L_*, grad T_{dim} are the dimensional value of the gravity acceleration, layer thickness, and temperature difference. This choice of Rayleigh number can be justified as follows: when all parameters except α_4, δ_1 are taken to be zero, the equations should describe convection in ordinary isotropic fluid. The Rayleigh number for monotonic instability is defined by the following expression:

$$Ra_{mon} = \frac{2m_{33}(m_{11}m_{22} - m_{12}m_{21})}{\alpha_4 \delta_1 k^2 (-m_{22} - \delta_2 m_{21})}. \tag{17}$$

The neutral monotonic instability curve $Ra_{mon} = Ra_{mon}(k)$ has a non-removable discontinuity at $k = k_d (Ra_{mon} > 0$ at $k < k_d$ and $Ra_{mon} < 0$ at $k > k_d)$. The value k_d is defined by condition of zero denominator in (17):

$$k_d{}^2 = \frac{\left(\delta_2(\gamma_2 - \gamma_1) + 2K_3 + 2\varepsilon_\alpha E_0{}^2\right) m^2 \pi^2}{\delta_2(\gamma_2 + \gamma_1) - 2K_1} \geq 0.$$

In particular, we have $k_d \approx 18.392$ for MBBA(1)and $k_d \approx 29.998$ for MBBA(2). The neutral monotonic instability curves $Ra_{mon} = Ra_{mon}(k)$ for MBBA in interval $0.1 \leq k \leq 35$ are shown in Figures 1, 2. The critical Rayleigh number Ra^*_{mon} (the minimal value of the Rayleigh number $Ra^*_{mon} = Ra^*_{mon}(k_*)$) and the corresponding wave number k_* for the layer heated from below are (see the curves 1 in Figs. 1, 2):

$$Ra^*{}_{mon} \approx 2.302, \ k_* \approx 2.291, \quad \text{for MBBA(1)},$$

$$Ra^*{}_{mon} \approx 2.310, \ k_* \approx 2.375, \quad \text{for MBBA(2)}.$$

The critical Rayleigh number $(-Ra^*_{mon})$ and the corresponding wave number k_*, for the MBBA layer heated from above are (see the curves 2 in Figs. 1, 2):

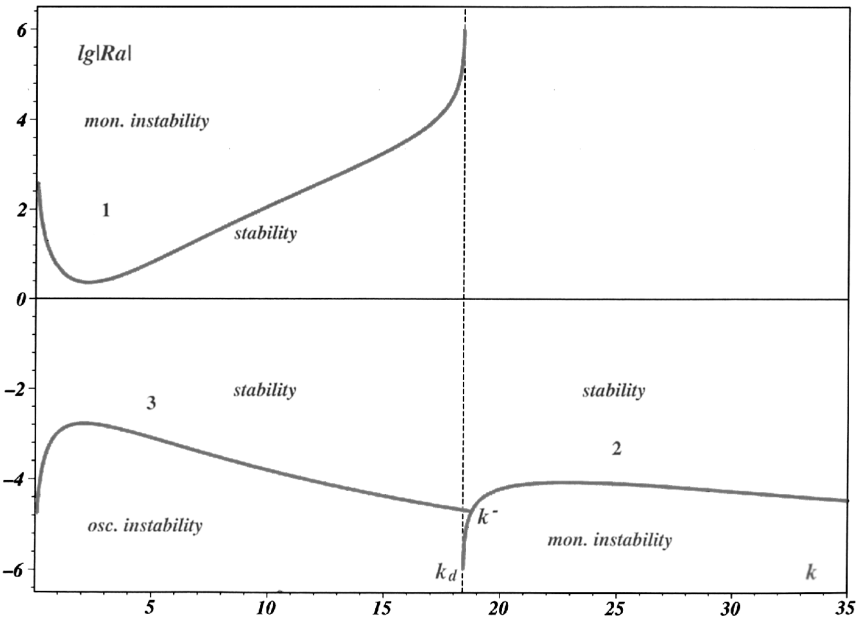

FIGURE 1 Neutral curves for MBBA(1).

$$Ra^*{}_{mon} \approx -11917.716, \ k_* \approx 22.838, \quad \text{for MBBA(1)},$$

$$Ra^*{}_{mon} \approx -184226.127, \ k_* \approx 36.939, \quad \text{for MBBA(2)}.$$

In Figures 1, 2 the function $1\mathrm{g}|Ra(k)|$ (not the function $Ra(k)$ itself) is shown. For the layer heated from below $Ra > 0$ and curve $1\mathrm{g}|Ra(k)|$ is shown in the upper half-plane. For the layer heated from above $Ra < 0$ and curve $1\mathrm{g}|Ra(k)|$ is formally presented in the lower half-plane (in order to avoid irrelevant intersections). Thus monotonic instability in liquid crystals can appear in both cases, whether the layer is heated from below or from above.

7. OSCILLATORY INSTABILITY

Dependence of the critical temperature gradient T'_0 for oscillatory instability on the parameters of the problem can be obtained via evaluation of the determinants of (14) at $\lambda = i\omega$, where ω is the oscillation frequency. Let us introduce the notation:

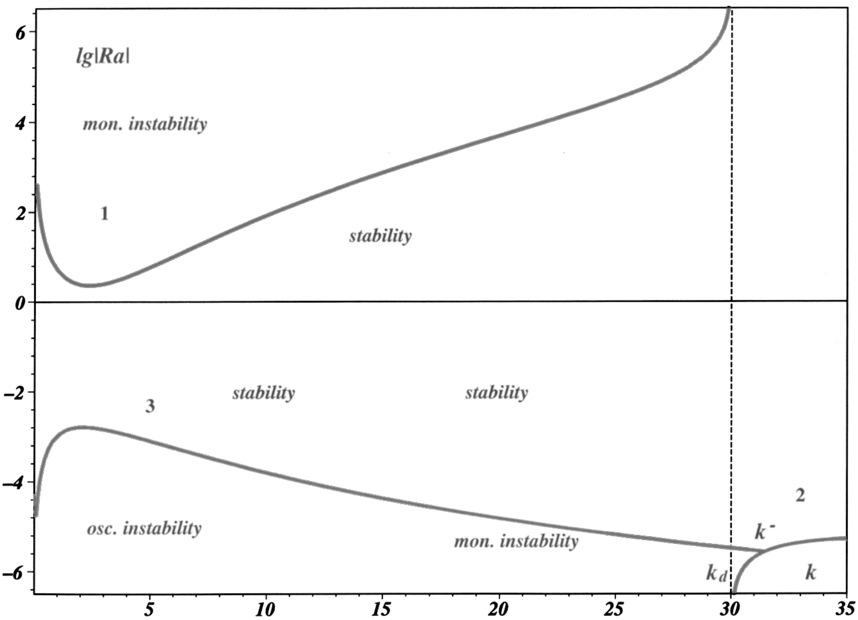

FIGURE 2 Neutral curves for MBBA(2).

$$m_3 = \gamma_1(m^2\pi^2 + k^2) > 0,\; m_1 = {m_1}^1 T_0' + {m_1}^0,\; {m_0}^1 T_0' + {m_0}^0,$$

$$\begin{aligned} m_2 = &-(\gamma_1 m_{11} - m_{22}(m^2\pi^2 + k^2)) - \gamma_1(m^2\pi^2 + k^2)m_{33} \\ &+ \frac{1}{2}(\alpha_2 + \alpha_3)m_{21}(-m^2\pi^2 + k^2),\; {m_0}^0 = (m_{11}m_{22} - m_{21}m_{22})m_{33}, \end{aligned}$$

$$\begin{aligned} {m_1}^0 &= -m_{11}m_{22} + m_{21}m_{12} - \frac{1}{2}(\alpha_2 + \alpha_3)m_{21}m_{33}(-m^2\pi^2 + k^2) \\ &= (\lambda_1 m_{11} - m_{22}(m^2\pi^2 + k^2))\, m_{33},\; {m_1}^1 = \gamma_1 k^2,\; {m_0}^1 = (\delta_2 m_{21} + m_{22})k^2. \end{aligned}$$

The Rayleigh number of oscillatory instability $Ra_{osc} = Ra_{osc}(k)$ and frequency ω are defined by the following expressions:

$$Ra_{osc} = \frac{2(m_3 m_0^0 - m_2 m_1^0)}{\alpha_4 \delta_1 (m_2 m_1^1 - m_3 m_0^1)},\; T_0' = \frac{1}{2}\alpha_4\delta_2 Ra_{osc},\; \omega^2 = \frac{(m_1^1 {T_0}' + m_1^0)}{m_3} > 0. \tag{18}$$

The neutral oscillatory instability curves $Ra_{osc} = Ra_{osc}(k)$ in the interval $0.1 \le k \le 35$ for MBBA are shown in Figures 1, 2 (see the curves 3). The critical Rayleigh numbers ($-Ra^*_{osc}$) and the corresponding wave numbers

k_* layer heated from above are

$$Ra^*_{osc} = Ra_{osc}(k_*),\ {\omega_*}^2 = \omega^2(k_*):$$
$$Ra^*_{osc} \approx -597.837,\ k_* \approx 2.098,\ \omega_*^2 \approx 1.18 \cdot 10^{-4}, \quad \text{for MBBA(1)}$$
$$Ra^*_{osc} \approx -622.463,\ k_* \approx 2.085,\ \omega_*^2 \approx 1.16 \cdot 10^{-4}, \quad \text{for MBBA(2)}$$

Note that $\omega^2 > 0$ is valid only if $0 < k < k^-$. It means that the oscillatory instability convection can appear only for $0 < k < k^-$. Note that the value k^- corresponds to the intersection of 'oscillatory' curve 3 and 'monotonic' curve 2. In this case (known as the 'intersection of bifurcations') one can expect the appearance of complicated secondary convective regimes. Calculations show that $k^- \approx 18.766$ for MBBA(1)and $k^- \approx 31.472$ for MBBA(2).

8. CONVECTIVE INSTABILITY OF THE MBBA

Let us choose the layer thickness $L_* = 0.005\,m$, and the gravity acceleration $g_* = 10\,m \cdot s^{-2}$. Then the critical temperature gradient corresponding to the monotonic instability for the heating from below is grad $T_{\text{dim}} = 0.012\,K$ (see (16)). The similar value for the oscillatory instability for the layer heated from above is grad $T_{\text{dim}} = -3.16\,K$. As we have already noted above, the values of critical parameters for MBBA(1) and MBBA (2) are practically coincide. This surprising fact requires further investigations. Imposing magnetic or electric fields in the case $\chi_a > 0, \varepsilon_a > 0$ does not change the qualitative behaviour of the neutral curves presented in Figures 1, 2. The increasing of the field intensity entails only the increasing of the critical numbers Ra^*. In the case when at least one of the parameters χ_a, ε_a is negative, a qualitative change of the behaviour of the neutral curves is possible. In particular, the neutral curves may cross the axis $Ra = 0$. That would mean the onset of usual hydrodynamic instability in the absence of any temperature gradients.

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